



Could the electroweak scale be linked to the large scale structure of the Universe?

Alak Chakravorty ¹ and
Alessandro Massarotti ^{1,2}

*Department of Physics
The University of Chicago ¹
Chicago, IL 60637 and
NASA / Fermilab Astrophysics Center
Fermi National Accelerator Laboratory ²
Batavia, IL 60510*

Abstract

We study a model where domain walls are generated through a cosmological phase transition involving a scalar field. We assume the existence of a coupling between the scalar field and dark matter and show that the interaction between domain walls and dark matter leads to an energy dependent reflection mechanism. For a simple Yukawa coupling, we find that the vacuum expectation value of the scalar field is $\langle \phi \rangle \simeq 30 \text{ GeV} - 1 \text{ TeV}$, in order for the model to be successful in the formation of very large scale “pancake” structures.



The existence of a very large scale structure of the Universe is one of the most interesting problems of modern Cosmology. The recent data of the so called “pencil beam” surveys,¹ scanning small portions of the sky up to redshifts close to unity, strongly suggests that clustering extends up to scales close to $100h^{-1}\text{Mpc}$. These data prompted new interest in scenarios for the formation of large scale structure alternative to the standard cold dark matter perturbation model.

Domain walls, generated through a second order cosmological phase transition, could be an avenue of research in this field, since they provide an alternative model for structure formation. In the first domain wall models, these topological defects were assumed to couple to the matter solely through gravitation.²⁻³ As a consequence of the weakness of the interaction, domain walls would stretch undisturbed under their surface tension and rapidly reach relativistic speeds, with the resulting network scale close to that of the horizon.⁴⁻⁵ Recent work by one of the authors⁶⁻⁷ has indicated that the assumption of a non-gravitational coupling between the walls and a cosmologically significant neutral component of dark matter can drastically change the features of the model. Briefly summarizing, it was shown that if such a coupling leads to particle reflection, then the domain walls can be slowed down very efficiently. At the same time, the walls can sweep large quantities of matter in their motion and, at a late stage of their evolution, give rise to wakes of $1 - 10h^{-1}\text{Mpc}$ thickness. The model also predicts comoving “interwake” distances of the order of $10 - 100h^{-1}\text{Mpc}$. It was found that fermions of mass $m \sim 1 - 10\text{eV}$ would be an ideal candidate for the dark matter interacting with the walls.

In this Letter, we will study the particle-kink interaction that derives from introducing a simple model coupling between the field of the walls ϕ and that of the dark matter ψ :

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda^2}{4}(\phi^2 - \phi_o^2)^2 + \bar{\psi}(i\partial)\psi + gf(\phi)\bar{\psi}\psi. \quad (1)$$

The scalar potential is chosen so as to give rise to a symmetry breaking, generating the domain walls and at least part of the fermion mass. Notice that any potential generating domain walls (e.g. a sine-Gordon potential) is well approximated by this “ ϕ^4 ” scalar Lagrangian. Our choice of the coupling is by no means exhaustive, but it gives us an opportunity to introduce the formalism appropriate to the study of the more general wall-particle scattering problem. We will not attempt to investigate the fundamental theory from which the \mathcal{L}_{eff} could originate.

We take the classical kink soliton solution for the scalar field.⁸⁻⁹ The field ϕ has vacuum expectation values $\langle\phi\rangle = +\phi_o$ or $-\phi_o$ and has the usual form $\phi_c(z) = \phi_o \tanh(z/\Delta)$, where Δ gives the thickness of the domain wall. In order to consider the behavior of the fermions as they pass through domain wall boundary we look at the stationary solutions to the Dirac equation, coming from the Lagrangian above,¹²

$$(\vec{\alpha} \cdot \vec{p} + g\beta f(\phi_c(z)))\psi = E\psi, \quad (2)$$

where $\psi = \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}$, ψ_a , ψ_b being two component spinors. We can write the Dirac equation (in the Dirac representation) as ¹⁰

$$E \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = \begin{bmatrix} -i\vec{\sigma} \cdot \vec{\nabla}\psi_b + g\psi_a f(\phi_c(z)) \\ -i\vec{\sigma} \cdot \vec{\nabla}\psi_a - g\psi_b f(\phi_c(z)) \end{bmatrix}. \quad (3)$$

Solving for ψ_b in terms of ψ_a we get:

$$E \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = \begin{bmatrix} -\vec{\sigma} \cdot \vec{\nabla} \left(\frac{\vec{\sigma} \cdot \vec{\nabla}(\psi_a)}{E + gf(\phi_c(z))} \right) + gf(\phi_c(z))\psi_a \\ -\vec{\sigma} \cdot \vec{\nabla} \left(\frac{\vec{\sigma} \cdot \vec{\nabla}(\psi_b)}{E - gf(\phi_c(z))} \right) - gf(\phi_c(z))\psi_b \end{bmatrix} \quad (4)$$

which is equivalent to

$$(E^2 - g^2 f^2(\phi_c)) \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = \begin{bmatrix} -\nabla^2 \psi_a + \frac{g \vec{\nabla} f(\phi_c) \cdot \vec{\nabla} \psi_a}{E + gf(\phi_c)} \\ -\nabla^2 \psi_b - \frac{g \vec{\nabla} f(\phi_c) \cdot \vec{\nabla} \psi_b}{E - gf(\phi_c)} \end{bmatrix}. \quad (5)$$

As one might expect there is no spin dependence and we can write the expression in one dimensional form:

$$-\frac{d^2 \psi_a}{dz^2} + \frac{g \frac{df(\phi_c)}{dz} \frac{d\psi_a}{dz}}{E + gf(\phi_c)} = (E^2 - g^2 f^2(\phi_c))\psi_a, \quad (6)$$

and a similar expression for ψ_b .

For particular choices of $f(\phi)$ one can simplify eq.(6). In particular, we studied the simple Yukawa case $f(\phi) = \phi$. The interaction term can be interpreted as a Majorana neutrino mass term, where we can use weak singlets to define ψ . If one

thinks of it as a Dirac neutrino mass, this term is clearly unaesthetic, since it breaks explicitly the SU_{2weak} symmetry of the interaction Lagrangian. Its presence should then be explained in the context of a larger theory in order to be realistic. Let us now consider the relative magnitude of the two terms on the l.h.s. of eqn.(6). Far away from the kink $d\phi_c/dz \rightarrow 0$, so this is just the Klein-Gordon equation for fermions of mass m , as we would expect. In the vicinity of the wall we can define a “momentum” $p_{eff} \equiv \sqrt{E^2 - m_{eff}^2(z)}$, where $m_{eff} = g\phi_c(z)$. The second term in eq.(6) is,

$$\frac{g \frac{d\phi_c}{dz} \frac{d\psi_a}{dz}}{E + g\phi_c} = \frac{g\phi_o \frac{d\psi_a}{dz} \cosh^{-2}(z/\Delta)}{\Delta(E + g\phi_c)} \simeq \frac{p_{eff}}{\Delta} O(1),$$

where $\max[g\phi_o/(E + g\phi_c)] = O(1)$. The first term in eq.(6) is, on the other end, of the order of p_{eff}^2 . The problem simplifies if, during the interaction with the kink, we can ignore the second term as compared to the first, i.e. if $p_{eff}^2 \gg p_{eff}\Delta^{-1}$, or $p_{eff} \gg \Delta^{-1}$. We can achieve this by imposing the condition $\Delta \gg m_\nu^{-1}$, since within the kink $p_{eff} \sim m_\nu = g\phi_o \gg \Delta^{-1}$. This turns out to be the condition we would like for the reflection of the fermions to occur without pair creation. In fact, the probability of this happening is approximately given by $P \sim (m/\Delta)^2 \exp(-\pi m\Delta)$, which is clearly suppressed by our condition on the barrier thickness.¹¹ Ignoring then the gradient term, we can write eq.(6) as:

$$-\frac{1}{2g^2\phi_o^2} \frac{d^2\psi_a}{dz^2} + \left(-\frac{E^2}{2g^2\phi_o^2} + \frac{1}{2} \tanh^2(z/\Delta) \right) \psi_a = 0, \quad (7)$$

or, with $m = g\phi_o$, we have

$$-\frac{1}{2m^2} \frac{d^2\psi_a}{dz^2} + \left(-\tilde{E} + \frac{1}{2} \tanh^2(z/\Delta) \right) \psi_a = 0 \quad (8)$$

This is a Schroedinger-like equation for a particle in a $V(z) = 1/2 \tanh^2(z/\Delta)$ “potential”, with “mass” $\tilde{m} = m^2$ and energy $\tilde{E} = E^2/2m^2$. This is a potential barrier with $\tilde{E} > V_o$. The reflection coefficient can be calculated exactly for this process and is given by¹⁴

$$R = \frac{\cosh^2(\pi m\Delta)}{\sinh^2(\pi E\Delta) + \cosh^2(\pi m\Delta)}. \quad (9)$$

Examining this reflection coefficient for different values of the fermion velocity, we find that clearly $R(v) \rightarrow 1$ as $v \rightarrow 0$ and $R(v) \rightarrow 0$ as $v \rightarrow 1$. In the intermediate range, i.e. for $m_\nu\Delta \gg v^{-2}$ but $v \ll 1$, we get

$$R = \frac{1}{\exp(\pi m_\nu v^2 \Delta) + 1} \quad (10)$$

This exponential behavior of the reflection coefficient in this regime was actually well approximated by a step function in ref.(7), in which we just supposed the existence of a threshold energy ϵ_o . Apart from the detailed knowledge of the reflection coefficient, eq.(10) tells us that the threshold energy is simply correlated to the thickness of the walls, since $\epsilon_o \sim \Delta^{-1}$.

A similar result could be obtained in an axion-like model, like the one explored in ref.(12). There, the fermion couplings with the axion field were written as $m_f e^{i\gamma_5 \phi / \phi_o}$, and the resulting Dirac equation contains a spin dependent fermion mass term $m^* = m_f + S_z \partial_z \phi_c$. The domain walls are a barrier or a well, of potential $\delta m^* \sim \Delta^{-1}$, for opposite helicity states.

For all the cases in which the threshold energy ϵ_o is correlated to the wall thickness as $\epsilon_o \sim \Delta^{-1}$, we can use the results of ref.(7) to determine the constants in the domain wall potential of eq.(1). In the model elaborated in ref.(7), “light” domain walls form some time well before recombination. The dynamics of the domain wall network is strongly influenced by the interaction of walls with the neutrino background. By being partly reflected from the walls, the fermions gives rise to a friction pressure P_f , which is a function of the wall speed v and of the temperature of the fermion gas. In the first stages of the wall evolution the friction turns out to be irrelevant. At a later stage, when the friction becomes dominant, the domain walls slow down and the comoving scale of the network at that epoch (which is roughly the scale of the horizon) remains frozen in. The comoving network scale at the freeze-in is typically $\bar{r} \sim 10 - 100 h^{-1} Mpc$ and the matter wakes are $1 - 10 h^{-1} Mpc$ thick, if $\epsilon_o \sim 10^{-4} - 10^{-5} eV$. The freeze in takes place at a redshift $z \sim 10^3 - 10^4$.

From eq.(10) we can infer that $\Delta \sim 10^4 - 10^5 eV^{-1}$. Since other dynamical considerations pin down the range for the wall surface density to $\sigma \sim 10^{-1} - 1 MeV^3$, using the values of σ and Δ we can determine all of the parameters in the Lagrangian. From the relations $\sigma \sim \lambda \phi_o^3$ and $\Delta^{-1} \sim \lambda \phi_o$ we conclude that $\lambda \sim 10^{-15} - 10^{-17}$, $\phi_o \sim 10^{1.5} - 10^3 GeV$. This statement would be correct in all models of interaction where one obtains $\epsilon_o \sim \Delta^{-1}$.

If one does not dismiss this concordance as a coincidence, the result may suggest a possible connection between the field ϕ , originating the topological defects, and the

scalar (Higgs) of the Weinberg-Salam model, which gives rise to the electroweak symmetry breaking (stable domain walls cannot arise from the doublet itself). Because of the small value of the λ coupling constant, the phase transition that gives rise to the domain walls actually takes place at much lower temperature than the electroweak scale. We can simply evaluate what is called the Ginzburg temperature by recalling that $T_g^4 \simeq \lambda^2 \phi_0^4$: we obtain $T_g \simeq 10^{3.5} \text{eV}$, which is the temperature at which stable domain walls actually form and begin stretching under their surface tension.

The smallness of the λ constant is typical of “light” domain wall models. Its value constitutes one of the outstanding problems to solve, since it is not easy to accomodate the cosmological constraints using the simplest “realistic” particle physics models available. More research is needed in this direction.

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